SOLUTION OF EXERCISE # 3.1

Exercise # 5.1

Q.1: Convert the following to Radian Measures.

(i) 120°

Sol. 120°

$$= 120 \times \frac{\pi}{180} = \boxed{\frac{2\pi}{3}\text{ rad}}$$

(ii) 540°

Sol. 540°

$$= 540 \times \frac{\pi}{180} = \boxed{3\pi\,\text{rad}}$$

(iii) $42^\circ\ 36'\ 12''$

Sol. $42^\circ\ 36'\ 12''$

$$= \left(42 + \frac{36}{60} + \frac{12}{3600}\right)^\circ$$

$$= (42 + 0.6 + 0.0033)^\circ$$

$$= 42.6033^\circ$$

$$= 42.6033 \times \frac{\pi}{180}$$

$$= \boxed{0.74\,\text{rad}}$$

(iv) $24^\circ\ 32'\ 30''$

Sol. $24°32'30''$

$$= \left(24 + \frac{32}{60} + \frac{30}{3600}\right)^\circ$$

$$= (24 + 0.5333 + 0.0083)^\circ$$

$$= 24.5416^\circ$$

$$= 24.5416 \times \frac{\pi}{180}$$

$$= \boxed{0.42\,\text{rad}}$$

Q.2: Convert the following to degree measure:

(i) $\frac{5\pi}{4}$ rad

Sol. $\frac{5\pi}{4}$ rad

$$= \frac{5\pi}{4} \times \frac{180}{\pi} = \boxed{225^\circ}$$

(ii) $\frac{2\pi}{3}$ rad

Sol. $\frac{2\pi}{3}$ rad

$$= \frac{2\pi}{3} \times \frac{180}{\pi} = \boxed{120^\circ}$$

(iii) 5.52 rad

Sol. 5.52

$$= 5.52 \times \frac{180}{\pi}$$

$$= \boxed{316°16'21''}$$

(iv) 1.30 rad

Sol. 1.30

$$= 1.30 \times \frac{180}{\pi}$$

$$= \boxed{74°29'4''}$$

Q.3: Find the missing elements ℓ, r, θ when:

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(i) $\ell = 8.4\text{cm}, \theta = 2.8\text{rad}$

Sol. Here $r = ?$

We know that: $\ell = r\theta$

$$\Rightarrow r = \frac{\ell}{\theta} = \frac{8.4}{2.8} = \boxed{3\text{cm}}$$

(ii) $\ell = 12.2\text{cm}, \ r = 5\text{cm}$

Sol. Here $\theta = ?$

We know that: $\ell = r\theta$

$$\Rightarrow r = \frac{\ell}{\theta} = \frac{12.2}{5} = \boxed{2.44\,\text{rad}}$$

(iii) $r = 620\text{m}, \quad \theta = 32^\circ$ (IIA-2019)

Sol. Here $\ell = ?$ $\quad \theta = 32^\circ = 32 \times \frac{\pi}{180} = 0.56$ rad

We know that: $\ell = r\theta = (620)(0.56) = \boxed{346.27\,\text{m}}$

Q.4: How far apart are two cities on the equator whose longitudes are 10º E and 50º W? (Radius of the earth is 6400km).

Sol. Here: $\ell = ?\ \theta = 10^\circ + 50^\circ = 60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3}$ rad & $r = 6400\text{km}$

We know that: $\ell = r\theta = (6400)\left(\frac{\pi}{3}\right) = \boxed{6702.06\,\text{km}}$

Q.5: A space man land on the moon and observers that the Earth's diameter subtends an angle of $1°54'$ at his place of landing. If the Earth's radius is 6400km, find the distance between the Earth and the Moon.

Sol. $\theta = 1°54' = \left[1 + \frac{54}{60}\right]^0 = 1.9^\circ = 1.9 \times \frac{\pi}{180} = 0.033$ rad

$\ell = 2 \times 6400\text{km} = 12800\text{km}$

$$r = \frac{\ell}{\theta} = \frac{12800}{0.033} = \boxed{387878.78\,\text{km}}$$

Q.6: The sun is about 1.496×10^8 km away from the Earth. If the angle subtended by the sun on the surface of the earth is 9.3×10^{-3} radians approximately. What is the diameter of the sun?

Sol. Here $r = 1.496 \times 10^8\text{km}$ & $\theta = 9.3 \times 10^{-3}$ rad

$\ell = r\theta = (1.496 \times 10^8)\ (9.3 \times 10^{-3})$

$\ell = 1391280\text{km} = \boxed{1.39 \times 10^6\,\text{km}}$

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Q.7: A horse moves in a circle, at one end of a rope 27cm long, the other end being fixed. How far does the horse move when the rope traces an angle of 70º at the center?

Sol. Here $\theta = 70^\circ = 70 \times \frac{\pi}{180} = \frac{7\pi}{18}$ rad & $r = 27\text{cm}$

$$\ell = r\theta = 27\left(\frac{7\pi}{18}\right) = \boxed{33\text{m}}$$

Q.8. Lahore is 68km from Gujranwala. Find the angle subtended at the center of the earth by the road, joining these two cities, Earth being regarded as a sphere of 6400km radius.

Sol. Here $\ell = 68\text{km}$, $r = 6400$, $\theta = ?$

$$\theta = \frac{\ell}{r} = \frac{68}{6400} = 0.010625\text{rad} = 0.010625 \times \frac{180}{\pi} = \boxed{36'32''}$$

Q.9: A circular wire of radius 6cm is cut straightened and then bend so as to lie along the circumference of a hoop of radius 24cm. Find the measure of the angle which it subtends at the center of the hoop. (IIA-2020)

Sol. Here $\ell = 2\pi(6) = 12\pi\text{cm}$, $r = 24\text{cm}$, & $\theta = ?$

$$\theta = \frac{\ell}{r} = \frac{12\pi}{24} = \frac{\pi}{2}\text{rad} = \frac{\pi}{2} \times \frac{180}{\pi} = \boxed{90^\circ}$$

Q.10: A pendulum 5 meters long swings through an angle of 4.5º, through what distance does the bob moves? (IA-2022)

Sol. Here $r = 5\text{m}$, $\theta = 4.5^\circ$ & $\ell = ?$

$$\theta = 4.5 \times \frac{\pi}{180} = 0.07854\text{ rad}$$

$$\ell = r\theta = 5(0.07854) = \boxed{0.39\text{ m}}$$

Q.11: A fly wheel rotates at 300 rev/min. If the radius is 6cm, through what total distance does a point on the rim travel in 30 seconds?

Sol. $w = 300$ rpm, $t = 30\text{sec} = 0.5\text{min}$ & $r = 6\text{cm}$

$\theta = 2\pi wt = 2\pi\,(300)\,(0.5) = 300\pi$

$\ell = r\theta = (6)\,(300\pi) = \boxed{5655\text{cm}}$ (IA-2018)